a survey on folded plate structures

SAMARTÍN, A. [1]**; MARTÍNEZ, J.** [2]

SUMMARY:

A compact formulation of the linear theory of folded plate structures utilizing matrix methods is given. Different usual approximations and comparison between them are also shown.

RÉSUMÉ:

On présente une formulation compacte sur la théorie linéaire des voiles minces pliés, à l'aide de méthodes de calcul matriciel des structures. En même temps, on indique différentes approximations à la théorie et on fait une comparaison entre les résultats.

RESUMEN:

Se presenta una formulación compacta de la teoría lineal de láminas plegadas utilizando métodos de cálculo matricial de estructuras. Se indican también diferentes aproximaciones a la teoría y se comparan algunos resultados.

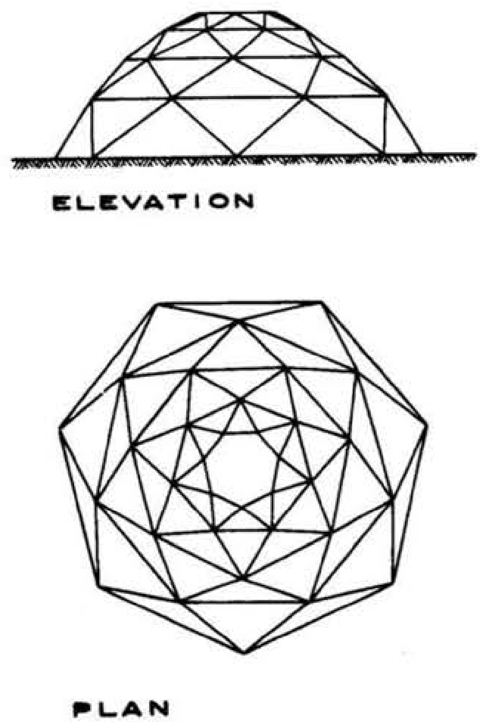
[1] Dr. Ing. Caminos, Lic. C. Matemáticas. D.I.C. Centro Estudios y Experimentación de O.P.

[2] Dr. Ing. Caminos. Centro de Estudios y Experimentación de O.P.

1.- DEFINITIONS AND HYPOTHESIS

1. 1. Definitions

A polyedrical shell (Fig. 1) is a shell having its middle surface composed of plans. Then it is posible, to distinguish in it, three types of --- structural elements: the node, the edge, and the plane.



POLYEDRICAL SHELL

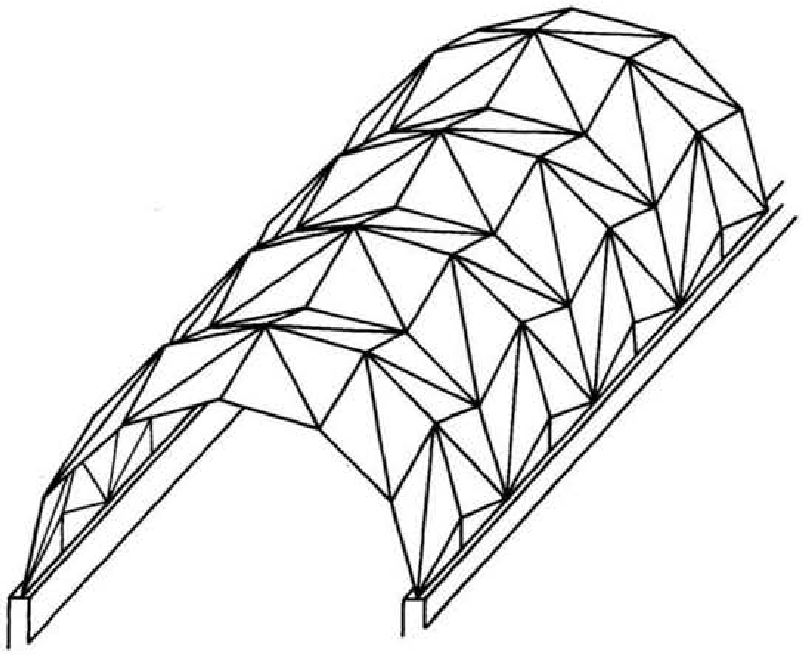
Fig. 1.

When the middle surface of the plyedrical shell is developable the --- structure is called folded plate. (Fig. 2).

A canonical folded plate structure is a folded plate structure without - internal nodes (i.e. nodes only along the boundaries). This structure can be defined by the following elements (Fig. 3):

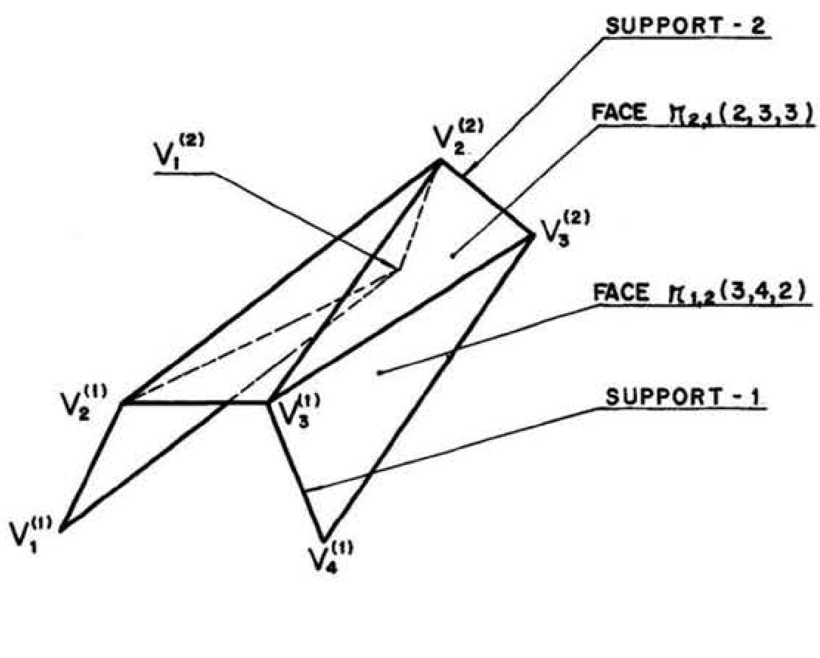
a) "Supports" or a set of m plan polygons $S^{(\alpha)}$, whose vertices are $V_i^{(\alpha)}$ $(i=1,2,--n_\alpha \quad \alpha=1,2,--m)$
b) "Faces" or a set of triangles $\pi_{\alpha,\beta}$ (i,j,k) defined by the vertices - $V_i^{(\alpha)}, V_j^{(\alpha)}, V_k^{(\beta)}$.

The supports are considered a part of the boundary of the folded plate - structure. If m = 2 the folded plate is called simple folded plate and -



FOLDED PLATE

Fig. 2.



CANONICAL FOLDED PLATE

Fig. 3.

continuous in the contrary case.

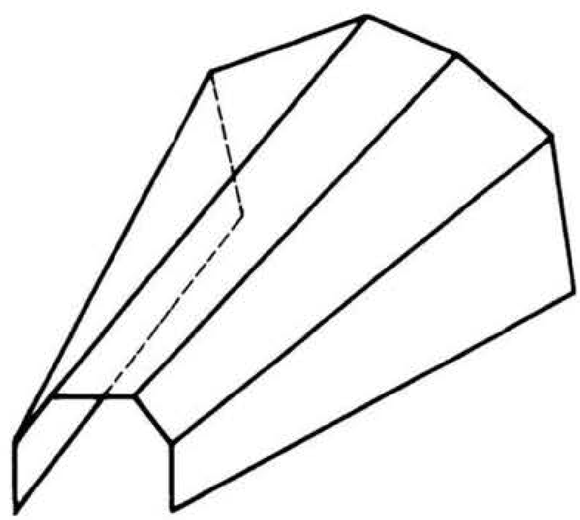
The following property defines the proper folded plate, as a particular case of the canonical folded plate:

For every face $\pi_{\alpha,\rho}(i,j,k)$ exists a vertex $V_l^{(\rho)}$ such as

$$\pi_{\alpha,\rho}(k,l,j) \equiv \pi_{\alpha,\rho}(i,j,l) \equiv \pi_{\rho,\alpha}(k,l,i) \equiv \pi_{\rho,\alpha}(k,l,j)$$

If the planes of the supports are paralell, the folded plate structure can be called normal. The span is defined, in this case, as the distance between plans of supports.

If the polygons of the support are equals, homothetic or without any special geometric relationship, the proper folded plate structures are divided accordingly in prismatic, conical and non-prismatic folded plates (Fig. 4).



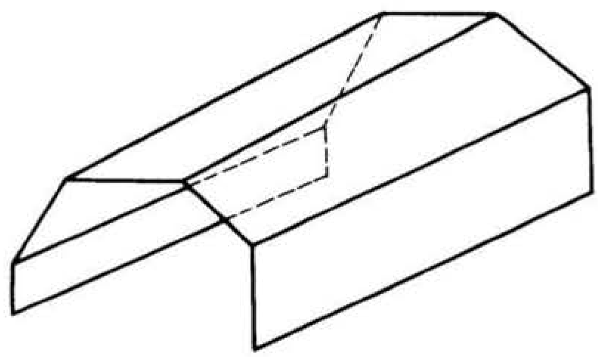
CONICAL FOLDED PLATE

Fig. 4.

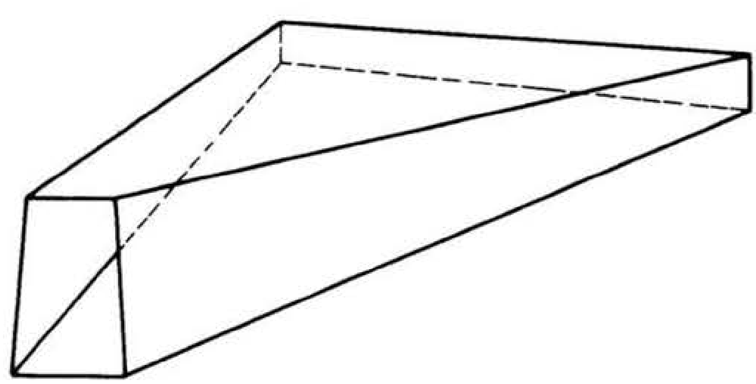
1. 2. Hypothesis

The hypothesis to be considered, corresponds to the linear elastic theory of shells,í, e.

a) Linear elastic material.
b) Small displacements, that means:
   b-1) To set up the static equations, the geometry of the shell corresponds to the undistorted shell.
   b-2) In the compatibility equations, the products including the displacements or its derivatives of any order are neglectible in comparison to the unity.



PRISMATIC FOLDED PLATE



NON-PRISMATIC FOLDED PLATE

Fig. 4.

c) The shell is thin, i.e., its structural behaviour can be defined by a bidimensional element (middle surface).
d) Kirchoff's assumptions.

The static-cinematic analogy of Goldenweizer is satisfied with the a) and b)hypothesis.

2. SIMPLE PRISMATIC FOLDED PLATE STRUCTURE, WITH NORMAL GABLE SUPPORTS

2. 1. Introduction

A exhaustive literature exists about this particular type of structures [1] and [2].

Frequentely, simplifications the hypotesis 1. 2. above stated, are made - in order to obtain an easy manual computation. But these analysis need - to be posible to loss some generality , introducing additional assumptions respect to either the geometry or the span/ width ratio of the structure.

A general and consistent analysis with the hypothesis 1. 2., is presented suitable to be used with a electronic computer.

2. 2. Displacements method

This matrix method of structural analysis presents a very simple -- formulation for a electronic computer [3] , in comparison with other - matrix methods. Examples: the compatibility method requires a knowledge - of algebraic topology to becomes a general approach and the transfer - matrix method, is used with advantage only in particular types of --- structures (branch structures) utilising matrix of minimum size, [5] and [6] .

In that follows, it is considered that every face is of constant - thickness and it is posible to divide its structural behaviour in two - independent parts: Membrane and flexural state.

The normal gable supports imply the following boundary conditions; for - each face:

$$u_2 = n_{11} = w = m_{11} = 0$$

Hence , a Levy type solution is posible. For the sake of clarity, the -- subscript m corresponding to the m-th term of the Fourier expansion is - dropped, in all the following formulas and only final results will be - presented. More details can be seen in [4].

2. 2. 1. Stiffness matrix.

2. 2. 1. 1. Flexural state.

From the general governing equation

$$\nabla^4 w = Z/D$$

it is obtained for every face:

$$\begin{bmatrix} \underline{p}_{s1} \\ \underline{p}_{s2} \end{bmatrix} = \underline{k}_s \begin{bmatrix} \underline{d}_{s1} \\ \underline{d}_{s2} \end{bmatrix} = \begin{bmatrix} \underline{k}_{s11} & \underline{k}_{s12} \\ \underline{k}_{s21} & \underline{k}_{s22} \end{bmatrix} \begin{bmatrix} \underline{d}_{s1} \\ \underline{d}_{s2} \end{bmatrix}$$

where

$$\underline{p}_s = \begin{bmatrix} r_2 \\ m_{22} \end{bmatrix}$$

$$\underline{d}_s = \begin{bmatrix} w \\ -w_{,2} \end{bmatrix} = \begin{bmatrix} w \\ \theta \end{bmatrix}$$

The elements of the matrices $\underline{P}_s$ and $\underline{d}_s$ are the coefficients of the ---

corresponding trigonometric functions.

The subscripts 1 and 2 refer to the edges 1 and 2. i.e., the edge $x_2 = 0$ and $x_2 = l_2$ (Figs. 5 and 6)

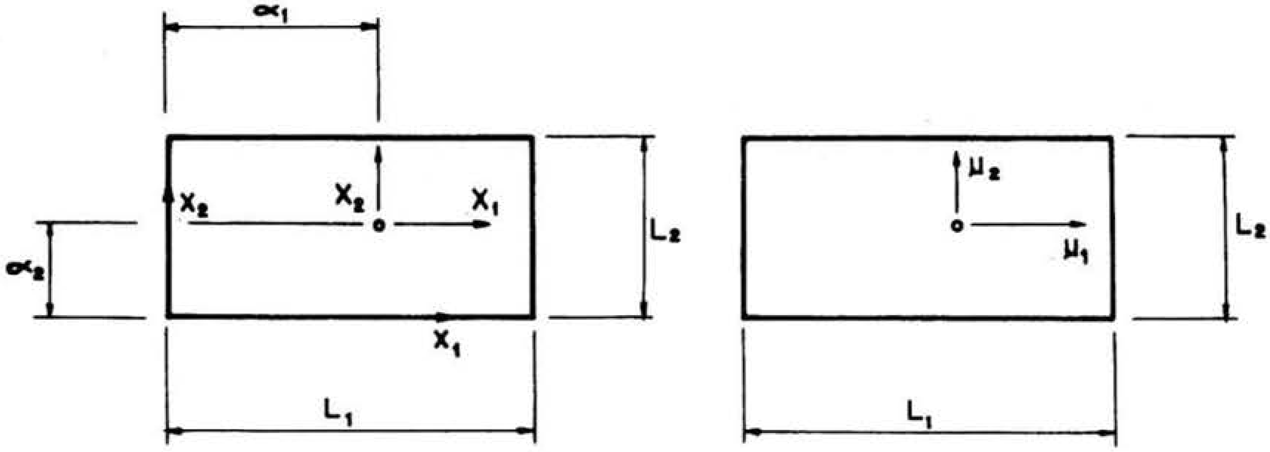
The stiffness matrix $\underline{K}_s$ is defined as:

$$\underline{k}_s = \underline{k}_{sp} \quad \underline{k}_{sd}^{-1}$$

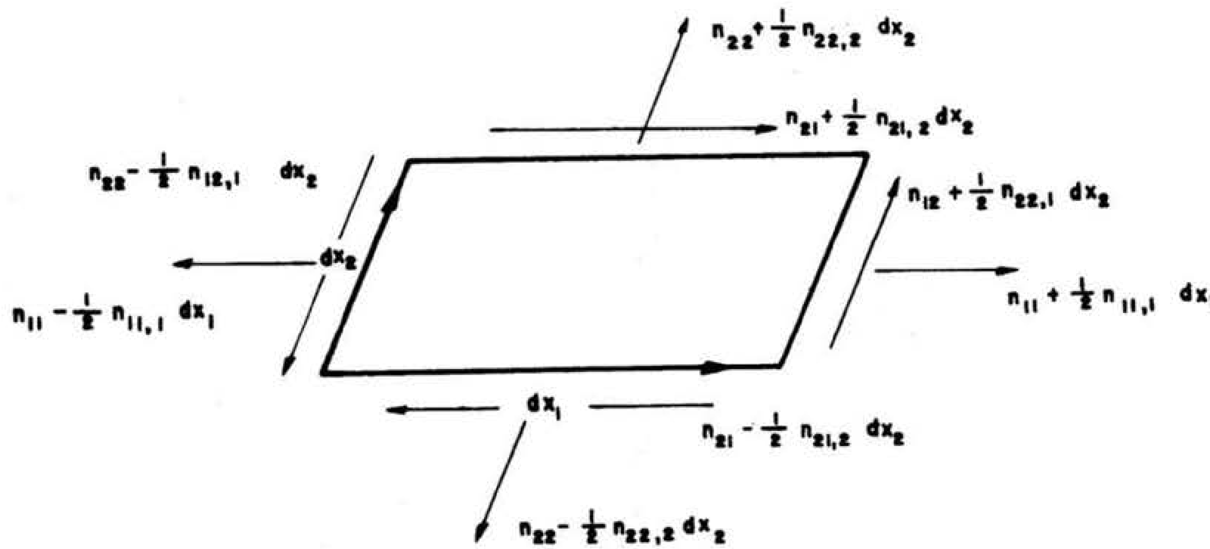
where

$$\underline{k}_{sp} = D \begin{bmatrix} (2-\nu)\lambda^2 \underline{B}^{(1)} - \underline{B}^{(3)} & -\left[(2-\nu)\lambda^2 \underline{B}^{(1)} - \underline{B}^{(3)}\right] \underline{P}(l_2) \\ \nu\lambda^2 \underline{B}^{(0)} - \underline{B}^{(2)} & \left[\nu\lambda^2 \underline{B}^{(0)} - \underline{B}^{(2)}\right] \underline{P}(l_2) \\ \left[(2-\nu)\lambda^2 \underline{B}^{(1)} - \underline{B}^{(3)}\right] \underline{P}(l_2) & -(2-\nu)\lambda^2 \underline{B}^{(1)} + \underline{B}^{(3)} \\ \left[\nu\lambda^2 \underline{B}^{(0)} - \underline{B}^{(2)}\right] \underline{P}(l_2) & -\nu\lambda^2 \underline{B}^{(0)} + \underline{B}^{(2)} \end{bmatrix}$$

$$\underline{k}_{sd} = \begin{bmatrix} \underline{B}^{(0)} & \underline{B}^{(0)} \; \underline{P}(l_2) \\ -\underline{B}^{(1)} & \underline{B}^{(1)} \; \underline{P}(l_2) \\ \underline{B}^{(0)} \; \underline{P}(l_2) & \underline{B}^{(0)} \\ -\underline{B}^{(1)} \; \underline{P}(l_2) & \underline{B}^{(1)} \end{bmatrix}$$



AXIS AND ACTIONS DISPLACEMENTS



STRESS - RESULTANTS

MEMBRANE

(Fig. 5)

$$\underline{B}^{(k)} = (-1)^k \left[\lambda^k, - k\lambda^{k-1}\right] \qquad (k=0,1,2 \text{ and } 3)$$

$$\underline{P}(l_2) = \begin{bmatrix} \exp(-\lambda l_2) & l_2 \exp(-\lambda l_2) \\ \bullet & \exp(-\lambda l_2) \end{bmatrix}$$

2. 2. 1. 2. Membrane state

Similary

$$\begin{bmatrix} \underline{p}_{m1} \\ \underline{p}_{m2} \end{bmatrix} = \underline{k}_m \begin{bmatrix} \underline{d}_{m1} \\ \underline{d}_{m2} \end{bmatrix}$$

where

$$\underline{p}_m = \begin{bmatrix} n_{12} \\ n_{22} \end{bmatrix}$$

and

$$\underline{d}_m = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

The subscripts 1 and 2 are used in the sense of 2. 2. 1. 1. (Fig. 5).

The stiffness matrix is

$$\underline{k}_m = \underline{k}_{mp} \; \underline{k}_{md}^{-1}$$

where

$$\underline{k}_{mp} = \begin{bmatrix} -\lambda \underline{B}^{(2)} & -\lambda \underline{B}^{(2)}\ \underline{P}(l_2) \\ -\lambda^2 \underline{B}^{(1)} & \lambda^2 \underline{B}^{(1)}\ \underline{P}(l_2) \\ -\lambda \underline{B}^{(2)}\ \underline{P}(l_2) & -\lambda \underline{B}^{(2)} \\ -\lambda^2 \underline{B}^{(1)}\ \underline{P}(l_2) & \lambda^2 \underline{B}^{(1)} \end{bmatrix}$$

$$k_{md} = \frac{1}{E(1-\nu^2)} \begin{bmatrix} -\nu\lambda \underline{B}^{(1)} - \frac{1}{\lambda} \underline{B}^{(3)} & \left[\nu\lambda \underline{B}^{(1)} + \frac{1}{\lambda} \underline{B}^{(3)}\right] \underline{P}(l_2) \\ -\lambda^2 \underline{B}^{(0)} - \nu \underline{B}^{(2)} & -\left[\lambda^2 \underline{B}^{(0)} + \nu \underline{B}^{(2)}\right] \underline{P}(l_2) \\ -\left[\nu\lambda \underline{B}^{(1)} + \frac{1}{\lambda} \underline{B}^{(3)}\right] \underline{P}(l_2) & \nu\lambda \underline{B}^{(1)} + \frac{1}{\lambda} \underline{B}^{(3)} \\ -\left[\lambda^2 \underline{B}^{(0)} + \nu \underline{B}^{(2)}\right] \underline{P}(l_2) & -\lambda^2 \underline{B}^{(0)} - \nu \underline{B}^{(2)} \end{bmatrix}$$

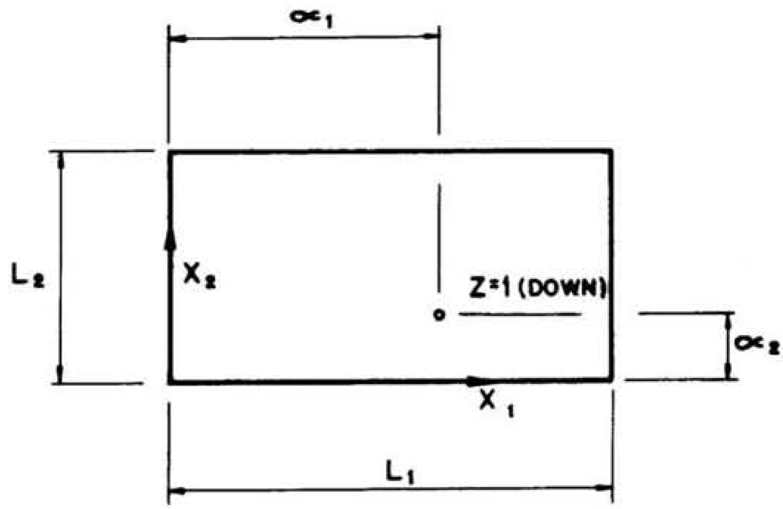
2. 2. 1. 3. <u>Total stiffness matrix</u>

From the above results, the stiffness matrix K, of the both states is:

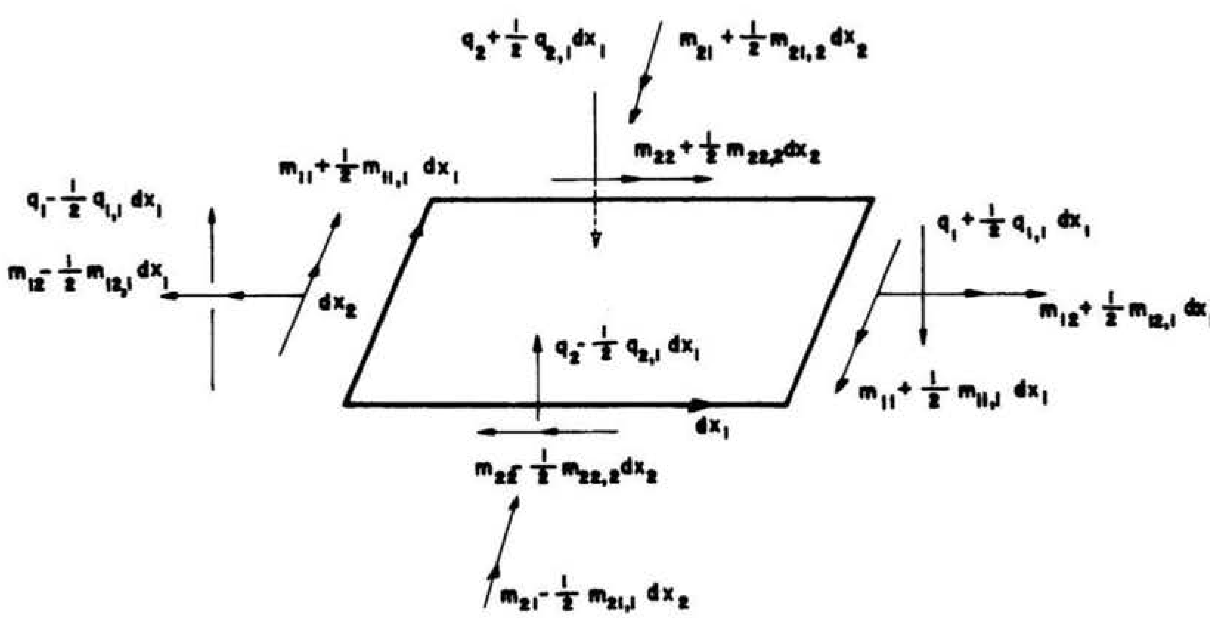
$$\underline{p} = \begin{bmatrix} \underline{p}_1 \\ \underline{p}_2 \end{bmatrix} = \begin{bmatrix} \underline{p}_{m1} \\ \underline{p}_{s1} \\ \underline{p}_{m2} \\ \underline{p}_{s2} \end{bmatrix} = \underline{k} \begin{bmatrix} \underline{d}_{m1} \\ \underline{d}_{s1} \\ \underline{d}_{m2} \\ \underline{d}_{s2} \end{bmatrix} = \underline{k} \begin{bmatrix} \underline{d}_1 \\ \underline{d}_2 \end{bmatrix} = \underline{k}\ \underline{d}$$

where

$$\underline{k} = \begin{bmatrix} \underline{k}_{11} & \underline{k}_{12} \\ \underline{k}_{21} & \underline{k}_{22} \end{bmatrix} \qquad \text{and} \qquad \underline{k}_{ij} = \begin{bmatrix} \underline{k}_{mij} & \cdot \\ \cdot & \underline{k}_{sij} \end{bmatrix} \qquad (ij = 1,\ 2)$$



AXIS AND ACTIONS



STRESS - RESULTANTS

PLATE

(Fig. 6)

2. 2. 2. Initial solution

2. 2. 2. 1. Definition

The stresses and displacements of the initial solution corresponds to the structure under the actual loads and null cinematic boundary conditions - along the edges 1 and 2 of every face.

2. 2. 2. 2. Flexural initial solution

In this case, by definition $\underline{d}_{s1} = \underline{d}_{s2} = 0$.

In the case of a punctual load Z_0, acting at the point (α_1, α_2)(Fig. 6) it is obtained:

$$\begin{bmatrix} \underline{p}_{s1} \\ \underline{p}_{s2} \end{bmatrix}_i = \begin{bmatrix} (\underline{k}_{sp})_{22}\, \underline{P}(\alpha_2)\, \underline{q}^{\circ} \\ (\underline{k}_{sp})_{11}\, \underline{P}(l_2-\alpha_2)\, \underline{q}^{\circ} \end{bmatrix} - \underline{K}_{sp}\, \underline{K}_{sd}^{-1} \begin{bmatrix} (\underline{k}_{sd})_{22}\, \underline{P}(\alpha_2)\, \underline{q}^{\circ} \\ (\underline{k}_{sd})_{11}\, \underline{P}(l_2-\alpha_2)\, \underline{q}^{\circ} \end{bmatrix}$$

where and similary

$$\underline{K}_{sp} = \begin{bmatrix} (\underline{k}_{sp})_{11} & (\underline{k}_{sp})_{12} \\ (\underline{k}_{sp})_{21} & (\underline{k}_{sp})_{22} \end{bmatrix}$$

$\underline{K}_{sd}$ have been defined in 2. 2. 1. 1.

$$\underline{P}(\alpha_2) = \begin{bmatrix} e^{-\lambda\alpha_2} & \alpha_2 e^{-\lambda\alpha_2} \\ \bullet & e^{-\lambda\alpha_2} \end{bmatrix}$$

and similary

$\underline{P}(l_2-\alpha_2)$

and

$$\underline{q}^{\circ} = \begin{bmatrix} 1 \\ \lambda \end{bmatrix} \frac{Z_0}{2\lambda^3} \quad \frac{\operatorname{sen}\lambda\alpha_1}{l_1 D}$$

2. 2. 2. 3. Membrane initial solution

It is possible to obtain the initial solution from a particular solution o the equation:

$$\nabla^4 \phi = \int (X^{\circ}_{1,22} + \nu X^{\circ}_{1,11})\, dx_1 + \int (X^{\circ}_{2,11} + \nu X^{\circ}_{2,22})\, dx_2$$

But usualy the punctual load X°_1, X°_2 is null and hence the initial solution.

is:

$$\begin{bmatrix} \underline{p}_{m1} \\ \underline{p}_{m2} \end{bmatrix}_i = \underline{0}$$

2. 2. 3. Results

With the above expressions of the stiffness matrix and the initial solution of every face, the standard tecnique of matrix methods of structural --- analysis can be used. [3]

A computer program 2 of this solution has been developed and some results - are given in the tables 1 and 2 for the folded plate shown in the Fig. 7

2. 3. Long folded plate structures. Simplifications.

2. 3. 1. Definition

When the diameter span ratio is small the folded plate can be called long. By diameter is means the maximum distance between two vertices of a --- support.

2. 3. 2. Moderate simplification

In a long folded plate the following assumptions can be introduced:

$$m_{11} = m_{12} = q_1 = r_1 = 0 \qquad \text{at every point of the}$$

shell.

These simplifications means a monodimensional flexural behaviour of the -- folded plate, [7] and the following results are summarized:

1^{st} - Flexural stiffness matrix:

$$k_s = \frac{E I_s}{l_2^3} \begin{bmatrix} -12 & 6l_2 & 12 & 6l_2 \\ 6l_2 & -4l_2^2 & -6l_2 & -2l_2^3 \\ -12 & 6l_2 & 12 & 6l_2 \\ -6l_2 & 2l_2^2 & 6l_2 & 4l_2^2 \end{bmatrix}$$

$$I_s = \frac{h^3}{12}$$

2nd - Membrane stiffness matrix

$$k_p = C^t \, k'_p \, C$$

where

$$C = \begin{bmatrix} 1 & \cdot & 1 & \cdot \\ 1 & \cdot & -1 & \cdot \\ \cdot & 1 & \cdot & 1 \\ \cdot & 1 & \cdot & -1 \end{bmatrix}$$

and

$$k'_p = \frac{Eh}{12} \begin{bmatrix} -3 l_2 \lambda^2 & \cdot & \cdot & \cdot \\ \cdot & l_2 \lambda^2 + \frac{6}{(1+\nu) l_2} & \cdot & \frac{-3\lambda}{1+\nu} \\ \cdot & \cdot & \frac{12}{l_2} & \cdot \\ \cdot & \frac{-3\lambda}{1+\nu} & \cdot & \frac{3 l_2}{2} \frac{\lambda^2}{1+\nu} \end{bmatrix}$$

3rd - Flexural initial solution

It is identical to a clamped-clamped beam along the edges 1 and 2 of the -- face.

4th - Membrane initial solution

It is assumed to be null.

The results obtained by a computer program for this solution are given in - the tables 1 and 2 for the structure shown in the Fig. 7.

Note.

The matrices K_s and K_p are non symetric because they link the displacements and stress-resultants instead of external actions.

2. 3. 3. <u>Strong simplification</u>

A consistent assumption with the static-cinematic analogy of Goldenweizer is the following one:

$m_{11} = m_{12} = q_1 = r_1 = 0$ (static assumption)

$e_{22} = e_{12} = 0$ (cinematic assumtion) (membrane strains l_{ij})

This simplification has often been used in manual computations.[1], and it is not developed here.

3. SIMPLE PRISMATIC FOLDED PLATE STRUCTURE WITH ARBITRARY SUPPORTS (HOMOGENOUS BOUNDARY CONDITIONS).

3. 1. Assumptions

The simplification 2. 3. 3. is adopted. The Poisson's ratio is null. ($\nu = 0$)

3. 2. Displacements method:

This method of analysis corresponds to a extension of the Levy solution, - using Rayleigh functions instead of trygonometric functions.

The Rayleigh functions $F_o^{(n)}(x_1)$ are a ordened set of orthogonal functions (eingenfunctions) corresponding to the eigenvalue problem: [8].

$$\begin{cases} \dfrac{d^4 F_o^{(n)}(x_1)}{dx_1^4} + \lambda_n^4 F_o^{(n)}(x_1) = 0 \qquad (n = 1, 2, \text{-----}, m, \text{-----}) \\ + \text{ Homogenous boundary conditions along the supports } x_1 = 0 \text{ and } x_1 = l_1 \end{cases}$$

For clarity, the subscript n, corresponding to the n- th Rayleigh function will be dropped down in all the following formulas, in similar way as the paragraph 2. 2.

The following notation will be used:

$$F_1(x_1) = \lambda \frac{d}{dx_1} F_o(x_1)$$

$$F_2(x_1) = \lambda \frac{d}{dx_1} F_1(x_1)$$

$$F_3(x_1) = \lambda \frac{d}{dx_1} F_2(x_1)$$

3. 2. 1. Stiffness matrix

3. 2. 1. 1. Flexural state

$$\begin{bmatrix} \underline{p}_{s1} \\ \underline{p}_{s2} \end{bmatrix} = \underline{k}_s \begin{bmatrix} \underline{d}_{s1} \\ \underline{d}_{s2} \end{bmatrix}$$

Where $\underline{P}_{si}$ and $\underline{d}_{si}$ have been defined in 2. 2. 1. 1. and $\underline{K}_s$ has the same -- expression as 2. 3. 2.

The elements of the matrices $\underline{P}_{si}$ and $\underline{d}_{si}$ are the coeficients of the $F_o(x_1)$.

3. 2. 1. 2. Membrane state

$$\begin{bmatrix} \underline{p}_{m1} \\ \underline{p}_{m2} \end{bmatrix} = \underline{k}_m \begin{bmatrix} \underline{d}_{m1} \\ \underline{d}_{m2} \end{bmatrix}$$

Where $\underline{P}_m$ and $\underline{d}_m$ are the matrices defined in 2. 2. 1. 2. and

$$\underline{K}_m = \underline{K}_{mp} \cdot K_{md}^{-1}$$

K_{mp} and K_{mp} have the same expression as the ones given in 2. 2. 1. 2. --- substituting in them $\nu = 0$ and

$$\underline{B}^{(k)} = \left[(-1)^k \lambda^k \cos\frac{k\pi}{4}, (-1)^{k+1} \lambda^k \operatorname{sen}\frac{k\pi}{4} \right] \qquad (K = 0,\ 1,\ 2, \ldots \text{ and } 3)$$

The variation of the elements of the matrices P_m and $\underline{d}_m$ along x_2 = constant is as follows:

u_1 like $F_1(x_1)$

u_2 and n_{22} like $F_o(x_1)$

n_{12} like $F_3(x_1)$

3. 2. 2. Initial solution

Is identical to the solution described in 2. 3. 2. A computer program has been developed using this analysis and some results are given in the table 3.

4. CANONICAL FOLDED PLATE STRUCTURES •ARBITRARY SUPPORTS

From the authors' point of vue the most convenient methods of analysis for this type of structures have to be found among open solutions [9] instead of analytical closed ones. And for this reason, a method of analysis using the finite element approach will be outlined here.

4. 1. Displacements method

The structural behaviour of every face, can be divided into both, above mentioned states.

4. 1. 1. Flexural state

The initial solution is represented (Fig. 8) by the vector

$$\underline{P}_{si} = \begin{bmatrix} \underline{P}_{s1} \\ \underline{P}_{s2} \end{bmatrix}_i$$

Every vector $(P_{s1})_i$ and $(P_{s2})_i$ is

$$\underline{P}_s = \begin{bmatrix} \underline{P}_s^{(1)} \\ \underline{P}_s^{(2)} \\ \underline{P}_s^{(ne)} \end{bmatrix}_i$$ along the edge 1 and 2 respect.

The column matrix $\underline{P}_s^{(j)}$ is defined as (j = 1, 2,... ne)

$$\underline{P}_s^{(j)} = \begin{bmatrix} r_2 \\ m_{22} \end{bmatrix}$$ at the point j.

Similary notation for $\underline{d}_s^{(j)}$.

The vector P_{si} can be obtained using any finite plate element, for example, the triangular plate element. The loads and the boundary conditions are -- known.

The stiffness matrix, is gives by the expression

$$\begin{bmatrix} \underline{P}_1 \\ \underline{P}_2 \\ \underline{o} \end{bmatrix} = \begin{bmatrix} \underline{k}_{11} & \underline{k}_{12} & \underline{k}_{1i} \\ \underline{k}_{21} & \underline{k}_{22} & \underline{k}_{2i} \\ \underline{k}_{j1} & \underline{k}_{j2} & \underline{k}_{ji} \end{bmatrix} \begin{bmatrix} \underline{d}_1 \\ \underline{d}_2 \\ \underline{d}_i \end{bmatrix}$$

Where the subscript i means now the internal points and the angular --- displacements or forces normal to the edge 1 and 2. The subscripts 1 and 2 are conected with the remaining displacements or forces along the edges 1 and 2.

$$\begin{bmatrix} \underline{P}_1 \\ \underline{P}_2 \end{bmatrix} = \underline{k} \begin{bmatrix} \underline{d}_1 \\ \underline{d}_2 \end{bmatrix}$$ where

$$\underline{k} = \begin{bmatrix} \underline{k}_{11} & \underline{k}_{12} \\ \underline{k}_{21} & \underline{k}_{22} \end{bmatrix} - \begin{bmatrix} \underline{k}_{1i} \\ \underline{k}_{2i} \end{bmatrix} \underline{k}_{ji}^{-1} \begin{bmatrix} \underline{k}_{j1} & \underline{k}_{j2} \end{bmatrix}$$

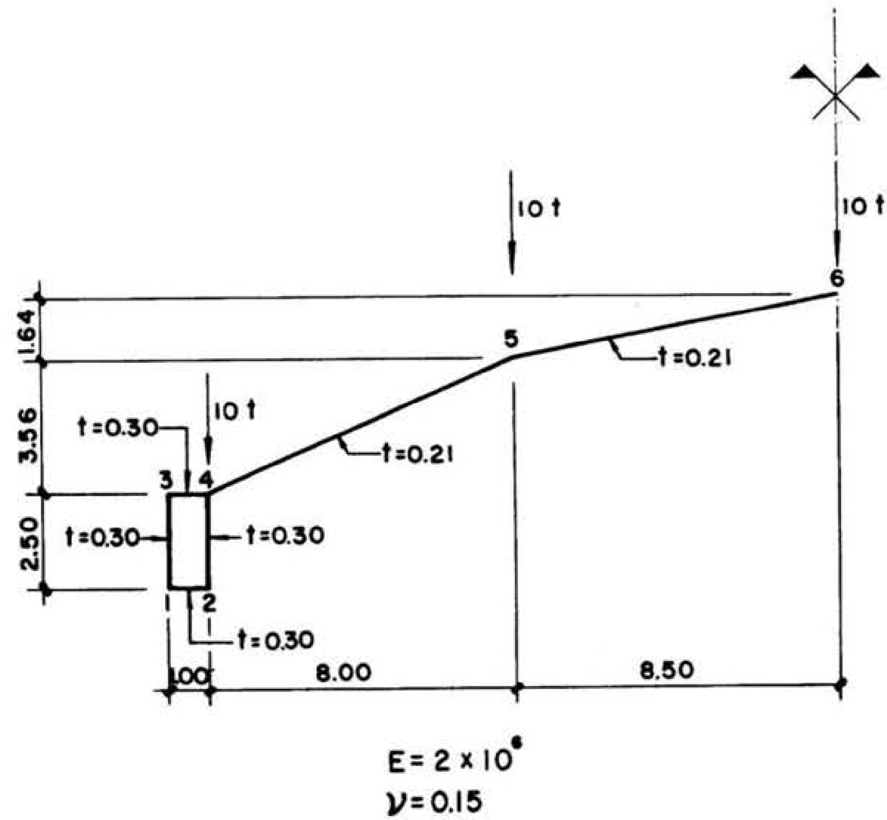

10 t
10 t
6
1.64
5
t=0.21
3.56
t=0.30
10 t
t=0.21
3
4
2.50
t=0.30
t=0.30
1
2
t=0.30
1.00
8.00
8.50
E = 2 x 10^6
ν = 0.15

Fig. 7.

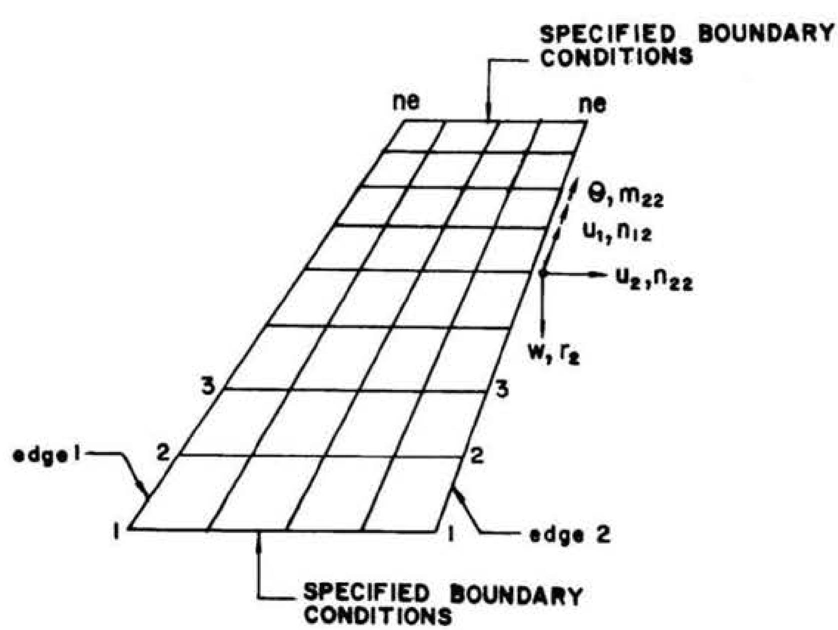

SPECIFIED BOUNDARY CONDITIONS
ne
ne
θ, m22
u1, n12
u2, n22
w, r2
3
3
2
2
edge 1
1
1
edge 2
SPECIFIED BOUNDARY CONDITIONS

Fig. 8.

4. 1. 2. Membrane state

Changing the subscript s by m, the stiffness matrix can be obtained in a similar way as in the last paragraph.

In this case the unknown displacements and forces are two by node instead of three as in the above case.

With the results of 4. 1. 1. and 4. 1. 2. the standard technical of matrix method of structural analysis could be used.

5. EXAMPLE

The example shown in the Fig. 7 has been analysed using the different -- aproachs previously described.

Results obtained by means an electronic computer are presented in the tables 1, 2, and 3.

The solution A is based in the analysis of the paragraph 2.3.

The solution B is obtained from the analysis of the paragraph 2. 2. --- assuming the faces 1-2, 1-3, 2-4 and 3-4 can be considered together as a box-beam with the following mechanical properties:

$I_2 = 0.4250 \ m^4$

$I_3 = 1.7300 \ m^4$

$J = 0.5350 \ m^4$

$S = 2.1000 \ m^2$

(For the notation see [4]).

The solution C is based in the analysis 2.2 without any simplification. - This solution can be considered "exact" and consistent with the linear - elasticity hypothesis 1.2.

TABLE NUMBER 1

"COMPARISON BETWEEN SOLUTIONS AND CONVERGENCE"

SPAN: 60 m.

LONGITUDINAL BOUNDARY CONDITIONS: SIMPLY SUPPORT/SIMPLY SUPPORT.

RESULT: $U2*10^3$ AT THE SECTION $X1 = L1/2$

SOLUTION		A			B			C		
NO. OF FOURIER TERMS		1	3	9	1	3	9	1	3	9
EDGE	1	94.4	94.7	94.6	91.1	91.2	91.2	92.3	92.4	92.4
	2	94.4	94.7	94.6	91.1	91.2	91.2	92.2	92.3	92.3
	3	95.7	95.7	95.7	91.7	92.1	92.1	92.9	92.9	92.9
	4	95.6	95.7	95.7	91.7	92.1	92.1	92.1	92.9	92.9
	5	44.7	42.7	43.1	42.2	40.3	40.7	43.1	41.2	41.5
	6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

RESULT: $W*10^3$ AT THE SECTION $X1 = L1/2$

SOLUTION		A			B			C		
NO. OF FOURIER TERMS		1	3	9	1	3	9	1	3	9
EDGE	1	259	245	246	250	249	250	255	254	254
	2	259	246	246	250	249	250	255	254	254
	3	259	246	246	250	249	250	255	254	254
	4	259	246	246	250	249	250	256	254	255
	5	146	135	135	141	135	136	144	138	139
	6	- 67.7	- 63.1	- 63.2	- 61.7	- 59.7	- 59.8	- 61.2	- 59.2	- 59.3

RESULT: $N11/h$ (Stress) AT THE SECTION $X1 = L1/2$

SOLUTION		A			B			C		
NO. OF FOURIER TERMS		1	3	9	1	3	9	1	3	9
EDGE	1	2536	2429	2446	2499	2386	2408	2492	2382	2401
	2	3057	2958	2975	3000	2899	2915	2994	2890	2908
	3	- 979	- 976	- 977	- 933	-1026	- 927	- 933	- 917	- 921
	4	- 440	- 437	- 434	- 432	- 413	- 419	- 415	- 400	- 405
	5	- 912	- 811	- 833	- 918	- 808	- 845	- 924	- 814	- 852
	6	- 196	- 284	- 263	- 263	- 331	- 324	- 261	- 330	- 323

TABLE NUMBER 1 (CONT)

RESULT: N12 AT THE SECTION XI = O

SOLUTION		A			B			C		
NO. OF FOURIER TERMS		1	3	9	1	3	9	1	3	9
EDGE	1	13.51	11.75	11.38	—	—	—	15.20	13.81	13.48
	2	30.41	37.03	40.25	—	—	—	27.86	34.29	37.97
	3	44.09	48.46	50.22	—	—	—	45.72	49.75	51.76
	4(4-2)	81.81	94.13	102.86	—	—	—	78.36	90.01	96.63
	4(4-3)	32.95	37.20	39.15	—	—	—	35.17	38.52	39.50
	4(4-5)	114.76	131.33	142.02	113.61	128.98	136.92	113.53	128.53	136.14
	5	49.72	51.43	52.39	51.11	53.68	55.40	51.41	54.13	56.00
	6	0.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00

RESULT: N22 AT THE SECTION XI = LI/2

SOLUTION		A			B			C		
NO. OF FOURIER TERMS		1	3	9	1	3	9	1	3	9
EDGE	1(1-2)	0.66	0.41	0.50	—	—	—	0.65	0.41	0.52
	1(1-3)	0.71	- 0.02	0.20	—	—	—	0.62	- 0.12	0.17
	2(2-1)	0.25	0.66	0.54	—	—	—	0.35	0.74	0.60
	2(2-4)	- 0.71	0.02	- 0.20	—	—	—	- 0.63	0.11	- 0.17
	3(3-1)	- 4.55	- 4.36	- 4.51	—	—	—	- 4.83	- 4.57	- 4.80
	3(3-4)	- 0.66	- 0.41	- 0.50	—	—	—	- 0.64	- 0.41	-0.51
	4(4-2)	- 9.56	- 4.70	- 6.90	—	—	—	- 9.04	- 4.28	- 6.44
	4(4-3)	- 2.64	- 1.71	- 1.95	—	—	—	- 2.73	- 1.91	- 2.22
	4(4-5)	- 2.74	- 1.19	- 1.60	- 2.67	- 1.12	- 1.85	- 2.62	- 1.22	- 1.72
	5(5-4)	-42.18	-24.80	-34.23	-42.22	-25.59	-33.99	-42.31	-25.69	-34.09
	5(5-6)	-38.58	-22.49	-31.26	-38.64	-23.30	-31.04	-38.73	-23.41	-31.15
	6(6-5)	-46.92	-35.17	-40.71	-47.49	-35.35	-40.93	-47.61	-35.46	-41.05

RESULT: M22 AT THE SECTION XI = LI/2

SOLUTION		A			B			C		
NO. OF FOURIER TERMS		1	3	9	1	3	9	1	3	9
EDGE	1	- 0.63	- 0.30	- 0.41	—	—	—	- 0.60	- 0.39	- 0.42
	2	0.08	- 0.33	- 0.12	—	—	—	0.01	- 0.28	- 0.24
	3	- 1.03	- 0.72	- 0.83	—	—	—	- 1.03	- 0.73	- 0.87
	4(4-2)	0.73	1.33	1.16	—	—	—	0.90	1.48	1.29
	4(4-3)	3.52	3.64	3.67	—	—	—	3.79	3.86	3.93
	4(4-5)	4.25	4.97	4.83	4.31	4.98	4.84	4.69	5.35	5.22
	5	6.85	5.94	6.10	6.60	5.65	5.91	6.59	5.63	5.89
	6	-18.63	-17.57	-17.74	-18.10	-17.02	-17.18	-18.30	-17.21	-17.37

TABLE NUMBER 1 (CONT)

RESULT M11 AT THE SECTION X1 = L1/2

SOLUTION		A			B			C		
NO. OF FOURIER TERMS		1	3	9	1	3	9	1	3	9
EDGE	1(1-2)	M 11 IS NULL BY HYPOTHESIS.			—	—	—	3.08	2.99	3.00
	1(1-3)				—	—	—	1.19	1.18	1.19
	2(2-1)				—	—	—	3.15	2.97	3.02
	2(2-4)				—	—	—	1.13	1.11	1.12
	3(3-1)				—	—	—	1.04	1.07	1.05
	3(3-4)				—	—	—	3.04	2.95	2.96
	4(4-2)				—	—	—	1.23	1.29	1.27
	4(4-3)				—	—	—	3.53	3.40	3.44
	4(4-5)				1.55	1.58	1.58	1.61	1.65	1.63
	5(5-6)				1.27	0.94	1.13	1.29	0.95	1.14
	6(6-5)				-2.06	-1.88	-1.88	-2.08	-1.90	-1.90

TABLE NUMBER 2

COMPARISON BETWEEN SOLUTIONS CONSIDERING THE VARIATION OF THE SPAN.

NUMBER OF FOURIER TERMS: 1

LONGITUDINAL BOUNDARY CONDITIONS: SIMPLY SUPPORT / SIMPLY SUPPORT.

RESULT: $W * 10^3$ AT THE SECTION X1 = L1/2

SPAN		60			30			15		
SOLUTION		A	B	C	A	B	C	A	B	C
EDGE	1	259	250	255	16.4	15.7	17.3	0.876	0.85	1.09
	2	259	250	255	17.3	17.0	18.1	1.05	1.01	1.27
	3	259	250	255	16.4	15.7	17.3	0.875	0.85	1.09
	4	259	250	256	17.4	17.0	18.2	1.09	1.01	1.31
	5	146	141	144	40.2	37.2	37.7	10.8	10.7	10.9
	6	-67.7	-61.7	-61.2	-19.5	-17.5	-17.4	-1.86	-1.64	-1.68

TABLE NUMBER 2 (CONT)

RESULT: N12 AT THE SECTION X1 = 0.

SPAN		60			30			15		
SOLUTION		A	B	C	A	B	C	A	B	C
EDGE	1	13.51	—	15.20	9.42	—	7.30	3.27	—	2.55
	2	30.41	—	27.86	32.62	—	30.67	13.81	—	13.84
	3	44.09	—	45.72	11.72	—	12.44	11.39	—	9.98
	4(4-2)	81.81	—	78.36	58.97	—	56.13	26.91	—	24.61
	4(4-3)	32.95	—	35.17	7.51	—	7.38	11.95	—	8.44
	4(4-5)	114.76	113.61	113.53	66.49	64.39	63.52	38.87	34.21	33.06
	5	49.72	51.11	51.41	14.38	16.12	16.52	2.49	5.31	5.78
	6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

RESULT: M22 AT THE SECTION X1 = L1/2

SPAN		60			30			15		
SOLUTION		A	B	C	A	B	C	A	B	C
EDGE	1	-0.63	—	-0.60	-0.95	—	-0.94	-0.95	—	-0.93
	2	0.08	—	0.01	1.42	—	1.35	1.12	—	1.10
	3	-1.03	—	-1.03	-0.78	—	-0.81	-0.96	—	-0.95
	4(4-2)	0.73	—	0.90	-2.30	—	-2.12	-1.48	—	-1.47
	4(4-3)	3.52	—	3.79	-1.34	—	-1.00	0.23	—	0.27
	4(4-5)	4.25	4.31	4.69	-3.64	-3.02	-3.13	-1.24	-1.27	-1.20
	5	6.85	6.60	6.59	5.44	5.18	5.21	1.48	1.83	1.84
	6	-18.63	-18.10	-18.30	-6.96	-6.70	-6.74	-1.65	-1.81	-1.84

RESULT M11 AT THE SECTION X1 = L1/2

SPAN		60			30			15		
SOLUTION		A	B	C	A	B	C	A	B	C
EDGE	1(1-2)	M11 IS NULL BY HYPOTHESIS	—	3.08	M11 IS NULL BY HYPOTHESIS	—	0.75	M11 IS NULL BY HYPOTHESIS	—	0.12
	1(1-3)		—	1.19		—	0.20		—	0.02
	2(2-1)		—	3.15		—	1.02		—	0.36
	2(2-4)		—	1.13		—	0.24		—	0.04
	3(3-1)		—	1.04		—	0.11		—	-0.13
	3(3-4)		—	3.04		—	0.77		—	0.12
	4(4-2)		—	1.23		—	-0.01		—	-0.18
	4(4-3)		—	3.53		—	0.79		—	0.28
	4(4-5)		1.55	1.61		-0.01	-0.00		-0.07	-0.04
	5(5-6)		1.27	1.29		1.17	1.18		0.94	0.95
	6(6-5)		-2.06	-2.08		-0.96	-0.96		-0.29	-0.29

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TABLE NUMBER 3: "STUDY OF THE EXAMPLE WITH DIFFERENT LONGITUDINAL BOUNDARY CONDITIONS"
SPAN: 60 m
NUMBER ØF FOURIER TERMS: 5

1. LONGITUDINAL BOUNDARY CONDITIONS: CLAMPED-CLAMPED

STRESS-RESULTANTS

		0.000•L		0.400•L		0.500•L		0.600•L		1.000•L	
		EDGE 1	EDGE 2	EDGE 1	EDGE 2	EDGE 1	EDGE 2	EDGE 1	EDGE 2	EDGE 1	EDGE 2
PLATE 1-2											
	M 22	-0.00	-0.00	0.63	0.97	0.74	1.09	0.63	0.97	-0.00	-0.00
	Q 2	-0.00	-0.00	1.61	1.61	1.84	1.84	1.61	1.61	-0.00	-0.00
	N 12	15.78	-65.64	3.72	-16.16	0.00	-0.00	-3.72	16.16	-15.78	65.64
	N 22	-0.00	0.00	0.46	-1.03	0.55	-1.14	0.46	-1.03	-0.00	0.00
	N 11/h	1769.51	2006.83	-787.98	-921.77	-901.33	-1064.66	-788.00	-921.79	1769.48	2006.80
PLATE 1-3											
	M 22	0.00	0.00	-0.63	-0.52	-0.74	-0.63	-0.63	-0.52	0.00	0.00
	Q 2	0.00	0.00	-0.46	-0.46	-0.55	-0.55	-0.46	-0.46	0.00	0.00
	N 12	-15.78	-26.43	-3.72	-3.55	-0.00	-0.00	3.72	3.55	15.78	26.43
	N 22	-0.00	0.00	1.61	0.46	1.84	0.31	1.61	0.46	-0.00	-0.00
	N 11/h	1769.51	-584.09	-787.98	306.80	-901.33	359.89	-788.00	306.80	1769.48	-584.09
PLATE 2-4											
	M 22	0.00	0.00	-0.97	-1.59	-1.09	-1.76	-0.97	-1.60	0.00	0.00
	Q 2	0.00	0.00	-1.03	-1.03	-1.14	-1.14	-1.03	-1.03	0.00	0.00
	N 12	65.64	-123.51	16.16	-30.64	0.00	-0.00	-16.16	30.64	-65.64	123.50
	N 22	0.00	0.00	-1.61	-11.58	-1.84	-13.72	-1.61	-11.58	0.00	0.00
	N 11/h	2006.83	-285.60	-921.77	154.09	-1064.66	175.81	-921.79	154.08	2006.80	-285.60
PLATE 3-4											
	M 22	-0.00	0.00	0.52	-0.98	0.63	-0.95	0.52	-0.98	-0.00	0.00
	Q 2	-0.00	-0.00	-0.46	-0.46	-0.31	-0.31	-0.46	-0.46	0.00	0.00
	N 12	26.43	-16.60	3.55	0.17	0.00	-0.00	-3.55	-0.17	-26.43	16.59
	N 22	0.00	0.00	-0.46	-0.60	-0.55	-0.86	-0.46	-0.60	0.00	0.00
	N 11/h	-584.09	-285.60	306.80	154.09	359.89	175.81	306.80	154.08	-584.09	-285.60
PLATE 4-5											
	M 22	-0.00	-0.00	2.58	6.14	2.71	6.54	2.58	6.14	-0.00	-0.00
	Q 2	-0.00	-0.00	0.99	0.99	1.05	1.05	0.99	0.99	-0.00	-0.00
	N 12	140.11	-34.76	30.47	-11.18	0.00	-0.00	-30.46	11.18	-140.10	34.76
	N 22	0.00	0.00	-2.23	-29.01	-2.66	-37.52	-2.23	-29.01	0.00	0.00
	N 11/h	-285.60	-908.15	154.09	327.91	175.81	372.68	154.08	327.93	-285.60	-908.12
PLATE 5-6											
	M 22	0.00	0.00	-6.14	-9.37	-6.54	-10.03	-6.14	-9.37	0.00	0.00
	Q 2	0.00	0.00	-1.90	-1.90	-2.02	-2.02	-1.90	-1.90	0.00	0.00
	N 12	34.76	-0.00	11.18	0.00	0.00	0.00	-11.18	-0.00	-34.76	0.00
	N 22	0.00	0.00	-26.25	-32.41	-34.14	-38.17	-26.26	-32.42	0.00	0.00
	N 11/h	-908.15	408.51	327.91	-80.09	372.68	-65.08	327.93	-80.11	-908.12	408.48

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TABLE NUMBER 3 (cont.)

2. LONGITUDINAL BOUNDARY CONDITIONS: CLAMPED-FREE

STRESS-RESULTANTS

		0.000*L		0.400*L		0.500*L		0.600*L		1.000*L	
		EDGE 1	EDGE 2	EDGE 1	EDGE 2	EDGE 1	EDGE 2	EDGE 1	EDGE 2	EDGE 1	EDGE 2
PLATE 1-2											
	M 22	-0.00	0.00	0.37	-0.30	0.23	-0.96	0.20	-1.53	-0.33	-4.43
	Q 2	-0.00	-0.00	0.06	0.06	-0.73	-0.73	-1.32	-1.32	-4.76	-4.76
	N 12	-127.05	43.89	-126.18	77.83	-117.96	78.50	-105.01	74.87	-0.00	-0.00
	N 22	-0.00	-0.00	0.48	0.67	0.47	1.53	0.58	2.34	0.68	6.24
	N 11/h	7681.43	8799.74	2474.50	2853.14	1667.68	1906.07	1021.16	1157.27	-0.00	-0.00
PLATE 1-3											
	M 22	0.00	0.00	-0.37	-0.82	-0.23	-0.96	-0.20	-1.25	0.33	-2.03
	Q 2	0.00	0.00	-0.48	-0.48	-0.47	-0.47	-0.58	-0.58	-0.68	-0.68
	N 12	127.05	-207.72	126.18	-171.51	117.96	-157.34	105.01	-137.29	0.00	-0.00
	N 22	-0.00	0.00	0.06	-4.78	-0.73	-7.34	-1.32	-10.41	-4.76	-22.69
	N 11/h	7681.43	-1174.88	2474.50	-192.24	1667.68	-62.09	1021.16	7.94	-0.00	-0.00
PLATE 2-4											
	M 22	-0.00	-0.00	0.30	1.38	0.96	2.87	1.53	4.34	4.43	11.16
	Q 2	-0.00	-0.00	0.67	0.67	1.53	1.53	2.34	2.34	6.24	6.24
	N 12	-43.89	-61.92	-77.83	13.24	-78.50	24.40	-74.87	32.46	0.00	-0.00
	N 22	0.00	0.00	-0.06	-4.92	0.73	-1.70	1.32	0.04	4.76	14.40
	N 11/h	8799.74	-49.91	2853.14	153.29	1906.07	151.13	1157.27	125.25	-0.00	-0.00
PLATE 3-4											
	M 22	-0.00	-0.00	0.82	3.95	0.96	6.38	1.25	9.15	2.03	20.65
	Q 2	-0.00	-0.00	4.78	4.78	7.34	7.34	10.41	10.41	22.69	22.69
	N 12	207.72	-199.16	171.51	-167.14	157.34	-155.28	137.29	-137.03	0.00	-0.00
	N 22	0.00	0.00	-0.48	-2.36	-0.47	-3.03	-0.58	-4.37	-0.68	-8.63
	N 11/h	-1174.88	-49.91	-192.24	153.29	-62.09	151.13	7.94	125.25	-0.00	-0.00
PLATE 4-5											
	M 22	0.00	-0.00	-5.34	4.36	-9.26	4.01	-13.49	3.42	-31.82	-0.43
	Q 2	0.00	0.00	-0.11	-0.11	-0.59	-0.59	-1.15	-1.15	-3.68	-3.68
	N 12	261.08	-129.89	153.90	-103.87	130.87	-90.43	104.57	-75.03	0.00	-0.00
	N 22	0.00	0.00	-1.79	-29.80	-1.37	-29.33	-1.70	-37.13	-0.98	-43.02
	N 11/h	-49.91	-3290.00	153.29	-975.71	151.13	-680.21	125.25	-423.75	-0.00	0.00
PLATE 5-6											
	M 22	0.00	0.00	-4.36	-16.94	-4.01	-21.64	-3.42	-26.35	0.43	-44.41
	Q 2	0.00	0.00	-2.60	-2.60	-3.14	-3.14	-3.64	-3.64	-5.38	-5.38
	N 12	129.89	0.00	103.87	0.00	90.43	0.00	75.03	0.00	0.00	-0.00
	N 22	0.00	0.00	-27.31	-35.35	-26.96	-37.83	-34.36	-44.47	-40.55	-54.30
	N 11/h	-3290.00	-2469.76	-975.71	-1380.55	-680.21	-995.09	-423.75	-671.51	0.00	-0.00

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TABLE NUMBER 3 (cont.)

3. LONGITUDINAL BOUNDARY CONDITIONS: SIMPLY SUPPORT-CLAMPED

STRESS-RESULTANTS

		0.000•L		0.400•L		0.500•L		0.600•L		1.000•L	
		EDGE 1	EDGE 2	EDGE 1	EDGE 2	EDGE 1	EDGE 2	EDGE 1	EDGE 2	EDGE 1	EDGE 2
PLATE 1-2											
	M 22	0.00	0.00	0.60	0.65	0.65	0.72	0.55	0.66	-0.00	-0.00
	Q 2	0.00	0.00	1.25	1.25	1.38	1.38	1.21	1.21	-0.00	-0.00
	N 12	6.07	-45.00	-0.33	2.55	-1.36	16.13	-2.61	28.94	-11.32	74.43
	N 22	0.00	0.00	0.49	-0.59	0.54	-0.66	0.44	-0.62	-0.00	0.00
	N 11/h	0.00	0.00	-1462.03	-1761.11	-1308.92	-1577.25	-938.96	-1117.35	2596.68	3026.23
PLATE 1-3											
	M 22	0.00	0.00	-0.60	-0.64	-0.65	-0.69	-0.55	-0.56	0.00	0.00
	Q 2	0.00	0.00	-0.49	-0.49	-0.54	-0.54	-0.44	-0.44	0.00	0.00
	N 12	-6.07	-23.84	0.33	1.70	1.36	7.74	2.61	13.02	11.32	39.18
	N 22	0.00	0.00	1.25	-1.05	1.38	-1.06	1.21	-0.64	-0.00	0.00
	N 11/h	0.00	0.00	-1462.03	614.17	-1308.92	548.85	-938.96	378.98	2596.68	-943.93
PLATE 2-4											
	M 22	0.00	0.00	-0.65	-0.81	-0.72	-0.93	-0.66	-0.90	0.00	0.00
	Q 2	0.00	0.00	-0.59	-0.59	-0.66	-0.66	-0.62	-0.62	0.00	0.00
	N 12	45.00	-90.06	-2.55	4.68	-16.13	33.12	-28.94	59.18	-74.43	147.53
	N 22	0.00	0.00	-1.25	-10.73	-1.38	-11.67	-1.21	-9.72	0.00	0.00
	N 11/h	0.00	0.00	-1761.11	294.92	-1577.25	262.03	-1117.35	185.23	3026.23	-452.06
PLATE 3-4											
	M 22	0.00	0.00	0.64	0.40	0.69	0.37	0.56	0.08	-0.00	-0.00
	Q 2	0.00	0.00	1.05	1.05	1.06	1.06	0.64	0.64	-0.00	-0.00
	N 12	23.84	-14.89	-1.70	1.14	-7.74	3.41	-13.02	5.04	-39.18	25.51
	N 22	0.00	0.00	-0.49	-1.04	-0.54	-1.20	-0.44	-0.87	0.00	0.00
	N 11/h	0.00	0.00	614.17	294.92	548.85	262.03	378.98	185.23	-943.93	-452.06
PLATE 4-5											
	M 22	0.00	0.00	0.41	6.62	0.55	6.57	0.81	5.99	-0.00	-0.00
	Q 2	0.00	0.00	0.80	0.80	0.81	0.81	0.77	0.77	-0.00	-0.00
	N 12	104.95	-32.74	-5.82	1.93	-36.54	14.65	-64.23	26.51	-173.04	50.76
	N 22	0.00	0.00	-2.14	-31.81	-2.40	-35.70	-1.98	-27.13	0.00	0.00
	N 11/h	0.00	0.00	294.92	525.68	262.03	476.34	185.23	355.34	-452.06	-1160.11
PLATE 5-6											
	M 22	0.00	0.00	-6.62	-12.65	-6.57	-12.41	-5.99	-10.98	0.00	0.00
	Q 2	0.00	0.00	-2.36	-2.36	-2.32	-2.32	-2.07	-2.07	0.00	0.00
	N 12	32.74	-0.00	-1.93	0.00	-14.65	-0.00	-26.51	-0.00	-50.76	0.00
	N 22	0.00	0.00	-28.85	-36.87	-32.48	-38.77	-24.55	-32.17	0.00	0.00
	N 11/h	0.00	0.00	525.68	11.23	476.34	2.56	355.34	-21.40	-1160.11	300.75

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TABLE NUMBER 3 (cont.)

4. LONGITUDINAL BOUNDARY CONDITIONS: SIMPLY SUPPORT-FREE

STRESS-RESULTANTS

		0.000*L		0.400*L		0.500*L		0.600*L		1.000*L	
		EDGE 1	EDGE 2	EDGE 1	EDGE 2	EDGE 1	EDGE 2	EDGE 1	EDGE 2	EDGE 1	EDGE 2
PLATE 1-2											
	M 22	0.00	0.00	0.19	0.23	0.17	0.16	0.05	0.00	-0.33	-0.31
	Q 2	0.00	0.00	0.42	0.42	0.34	0.34	0.05	0.05	-0.65	-0.65
	N 12	5.24	-29.42	-1.64	4.71	-1.79	8.77	-1.59	11.17	0.00	-0.00
	N 22	0.00	0.00	0.15	-0.21	0.15	-0.13	0.05	0.02	-0.29	0.25
	N 11/h	0.00	0.00	-749.71	-898.41	-653.27	-792.53	-498.17	-608.60	0.00	0.00
PLATE 1-3											
	M 22	0.00	0.00	-0.19	-0.19	-0.17	-0.20	-0.05	-0.08	0.33	0.39
	Q 2	0.00	0.00	-0.15	-0.15	-0.15	-0.15	-0.05	-0.05	0.29	0.29
	N 12	-5.24	-15.34	1.64	1.75	1.79	3.29	1.59	4.92	-0.00	0.00
	N 22	0.00	0.00	0.42	-0.22	0.34	-0.46	0.05	-0.38	-0.65	0.88
	N 11/h	0.00	0.00	-749.71	309.93	-653.27	280.60	-498.17	220.26	0.00	-0.00
PLATE 2-4											
	M 22	0.00	0.00	-0.23	-0.31	-0.16	-0.16	-0.00	0.06	0.31	0.32
	Q 2	0.00	0.00	-0.21	-0.21	-0.13	-0.13	0.02	0.02	0.25	0.25
	N 12	29.42	-57.46	-4.71	7.97	-8.77	16.65	-11.17	22.24	0.00	-0.00
	N 22	0.00	0.00	-0.42	-3.70	-0.34	-3.16	-0.05	-0.88	0.65	5.74
	N 11/h	0.00	0.00	-898.41	150.11	-792.53	133.32	-608.60	105.04	0.00	-0.00
PLATE 3-4											
	M 22	0.00	0.00	0.19	0.03	0.20	0.26	0.08	0.29	-0.39	-0.49
	Q 2	0.00	0.00	0.22	0.22	0.46	0.46	0.38	0.38	-0.88	-0.88
	N 12	15.34	-10.62	-1.75	1.35	-3.29	1.50	-4.92	2.37	-0.00	0.00
	N 22	0.00	0.00	-0.15	-0.17	-0.15	-0.37	-0.05	-0.17	0.29	0.82
	N 11/h	0.00	0.00	309.93	150.11	280.60	133.32	220.26	105.04	-0.00	-0.00
PLATE 4-5											
	M 22	0.00	0.00	0.28	2.71	-0.09	1.88	-0.36	0.84	0.17	-3.13
	Q 2	0.00	0.00	0.34	0.34	0.20	0.20	0.05	0.05	-0.33	-0.33
	N 12	68.08	-18.02	-9.32	1.40	-18.16	5.84	-24.61	8.65	0.00	-0.00
	N 22	0.00	0.00	-0.58	-9.42	-0.64	-10.34	-0.18	-3.32	1.32	19.37
	N 11/h	0.00	0.00	150.11	275.62	133.32	227.55	105.04	161.25	-0.00	-0.00
PLATE 5-6											
	M 22	0.00	0.00	-2.71	-5.07	-1.88	-3.81	-0.84	-2.04	3.13	6.38
	Q 2	0.00	0.00	-0.95	-0.95	-0.69	-0.69	-0.35	-0.35	1.16	1.16
	N 12	18.02	0.00	-1.40	0.00	-5.84	-0.00	-8.65	-0.00	0.00	-0.00
	N 22	0.00	0.00	-8.44	-13.06	-9.41	-11.33	-3.00	-4.58	17.69	20.07
	N 11/h	0.00	0.00	275.62	-4.69	227.55	17.27	161.25	31.67	-0.00	0.00

6. CONCLUSIONS

The solutions A, B and C applied to above example give very closed results, because the maximum difference among them is less than 10%.

From the table I is drawing the speed of the convergence is nearly the same in the three solutions. In general the three first terms of Fourier expansion give enough accuracy for practical purposes in comparison with the nine - first terms. This speed of the convergence in the results is extremely good in the displacements, but worst in the membrane stress-resultants.

The simplifications $m_{11} = 0$ can be used in order to obtain the remaining results, but same times will be necessary to take into account the actual value m_{11}.

The table II shows a slight slow convergence of the results and more pronounced differences among the solution A, B and C as the span decreases.

It is presented in the table III results corresponding to the shell of the Fig. 7 with several longitudinal boundary conditions. The convergence of the results is good and comparison with other solutions (open solution) is now in progress.

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NOTATION

x_1 x_2 z	cartesian coordinate system
u_1 u_2 w	displacements
X_1 X_2 Z	loading actions
n_{ij}	membrane stress resultants
m_{ij}	flexural stress resultants
q_i	shear stress-resultants
r_i	Kirchoff shear
l_i	lengthness of a face
h	thickness of a face
ν	Poisson's ratio
$D = \dfrac{E\ h^3}{12(1-\nu^2)}$	
$\underline{P}$	force vector
$\underline{d}$	displacement vector
$\underline{K}$	stiffness matrix
$\lambda = \dfrac{n\ \pi}{l_1}$ $(n = 1, 2, \ldots)$.	

SUBSCRIPTS

m	refers to the membrane state
s	refers to the flexural state
o	refers to the initial solution
,i	means derivative respect to the variable x_i.

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